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$$y-ae^{2x}-be^{-3x}-ce^{x}=0.$$

2°. If the problem be generalized and the complete primitive taken in the form

$$y-ae^{mx}-be^{-(m+n)x}-ce^{nx}=0,$$

the corresponding differential equation of the third order is readily found to be

$$y'''-(m^2+mn+n^2)y'+mn(m+n)y=0.$$

The values $m=2$ and $n=1$ give the original problem.

3°. If the problem be completely generalized and the original primitive taken in the form

$$y-ae^{px}-be^{qx}-ce^{rx}=0,$$

the differential equation is

$$y'''-(p+q+r)y''+(pq+qr+rp)y'-pqry=0.$$

Putting $p+q+r=0$ we have the second case above. If in addition to $p+q+r=0$, $p=2$ and $q=1$, the first particular case appears again.

II. Solution by **WALTER HUGH DRANE, A. M.**, Professor of Mathematics, Jefferson Military College, Washington, Miss.

(1) $y=c_1e^{2x}+c_2e^{-3x}+c_3e^{x}$. Differentiate (1).

(2) $\dfrac{dy}{dx}=2c_1e^{2x}-3c_2e^{-3x}+c_3e^{x}$. Subtract (1) from (2).

(3) $\dfrac{dy}{dx}-y=c_1e^{2x}-4c_2e^{-3x}$. Differentiate (3).

(4) $\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}=2c_1e^{2x}+12c_2e^{-3x}$. Subtract twice (3) from (4).

(5) $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=20c_2e^{-3x}$. Differentiate (5).

(6) $\dfrac{d^3y}{dx^3}-3\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}=-60c_2e^{-3x}$. Add 3 times (5) to (6).

(7) $\dfrac{d^3y}{dx^3}-7\dfrac{dy}{dx}+6y=0$. Q. E. D.

See Johnson's Differential Equations, page 104, example 7.

MECHANICS.

61. Proposed by **WILLIAM HOOVER, A. M., Ph. D.**, Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

A body is suspended from a fixed point by an elastic string, which is stretched to double its natural length when the body is in equilibrium. Find how much the body must be depressed, so that when let go, it may just reach the point of suspension.

I. Solution by the PROPOSER.

Let a', a be the stretched and unstretched lengths of the string. T=the tension, λ=the modulus of elasticity, W=the weight attached, and x=the distance of the latter from the point of suspension at any time t from the beginning of motion.

By Hooke's law, $a'=a[1+(T/\lambda)]$ ....(1).

By the problem, when $a'=2a$, $T=W$, and (1) gives $\lambda=W$.

The forces acting are W and T acting downward and upward ; then

$$\frac{W}{g}\frac{d^2x}{dt^2}=W-\frac{W(x-a)}{a} \qquad \text{.....................(2).}$$

Multiplying both sides of (2) by $2(dx/dt)$ and integrating,

$$\frac{dx^2}{dt^2}=\frac{g}{a}(4ax-x^2)+C \qquad \text{.......................(3).}$$

When $x=x'$, $(dx/dt)=0$, and (3) gives $C=-(g/a)(4ax'-x'^2)$, and (3) is

$$\frac{dx^2}{dt^2}=\frac{g}{a}[(4ax-x^2)-(4ax'-x'^2)] \qquad \text{..................(4).}$$

When the body next comes to rest, $(dx/dt)=0$, and $x=0$, giving $4ax'-x'^2=0$, or $x'=4a$, or $x'=0$. $C=0$ in (3) gives

$$\sqrt{\left(\frac{g}{a}\right)}dt=\frac{dx}{\sqrt{4ax-x^2}} \qquad \text{....................(5).}$$

$$\text{Integrating, } t=\left[\sqrt{\left(\frac{a}{g}\right)}\text{versin}^{-1}\left(\frac{x}{2a}\right)\right]_0^{4a}=\pi\sqrt{\left(\frac{a}{g}\right)} \qquad \text{..............(6),}$$

the *time* for the motion.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

In what follows we neglect the weight of the string, assume Hooke's Law, that the tention of the string is proportional to its extension beyond the natural length, holds throughout the motion.

Let W=weight of body, l=natural length of string, a=extension due to weight W, b=extension at the time body begins to rise after depression, x=extension at any time t, T=corresponding tension of string.

Then by Hooke's Law, $T=Wx/a$ ....(1).

The differential equation of motion is

$$m\frac{d^2x}{dt^2}=W-T. \quad \text{But } W=Mg,\ T=Mgx/a. \quad \therefore\ \frac{d^2x}{dt^2}=(g/a)(a-x) \ \text{......(2).}$$

$$\left(\frac{dx}{dt}\right)^2=(g/a)(2ax-x^2)+B. \quad \text{When } t=0,\ x=b,\ \frac{dx}{dt}=0,\ B=(g/a)(b^2-2ab).$$

$$\therefore\left(\frac{dx}{dt}\right)^2=(g/a)(2ax-x^2+b^2-2ab)=v^2.$$

When $x=0$, $v=\sqrt{\frac{gb}{a}(b-2a)}$ = velocity of projection, h = height of projection $=\frac{v^2}{2g}=\frac{b}{2a}(b-2a)$, but $h=2l$ and $a=l$.

$$\therefore\ 2l=(b/2l)(b-2l) \text{ or } b=l(1+\sqrt{5}).$$

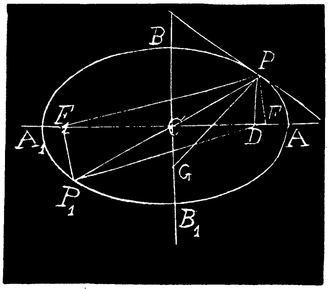
62. **Proposed by J. SCHEFFER, A. M., Hagerstown, Md.**

A particle of mass m moves in the circumference of an ellipse with constant rate v. It is constrained to move in that circumference by attractive forces in the two foci. To determine the magnitude of these forces,

I. **Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.**

Let P be the particle mass m, $CA=a$, $CB=b$, $PF=r$, $PF_1=r_1$, $CD=x$, $PD=y$, force along $r=f$, force along $r_1=f_1$. R= normal reaction, X=component of f and f_1 parallel to AC, Y=component of f and f_1 parallel to CB, $x^2/a^2+y^2/b^2=1$, the equation to the ellipse. The equations of motion are

$$m(d^2x/dt^2)=X-R(dy/ds)\,;\ m(d^2y/dt^2)$$
$$=Y+R(dx/ds)\ldots\ldots(1,2).$$



Now $ds/dt=v$, $\dfrac{dx}{ds}=\dfrac{a^2y}{\sqrt{a^4y^2+b^4x^2}}=\dfrac{ay}{b\sqrt{rr_1}}$, $\dfrac{dy}{ds}=\dfrac{b^2x}{\sqrt{a^4y^2+b^4x^2}}=\dfrac{bx}{a\sqrt{rr_1}}$

$$\left(\frac{dx}{dt}\right)^2=\left(\frac{ds}{dt}\right)^2\frac{a^4y^2}{a^4y^2+b^4x^2}=\frac{a^2v^2y^2}{b^2(a^2-e^2x^2)}=\frac{v^2(a^2-x^2)}{a^2-e^2x^2}.$$

$$\left(\frac{dy}{dt}\right)^2=\left(\frac{ds}{dt}\right)^2\cdot\frac{b^4x^2}{a^4y^2+b^4x^2}=\frac{b^2v^2x^2}{a^2(a^2-e^2x^2)}.$$

$$\therefore\ \frac{d^2x}{dt^2}=-\frac{a^2v^2x(1-e^2)}{(a^2-e^2x^2)^2}=-\frac{b^2v^2x}{r^2r_1^2},\ \frac{d^2y}{dt^2}=\frac{a^2v^2y}{(a^2-e^2x^2)^2}=\frac{a^2v^2y}{r^2r_1^2}.$$

$$R=(f+f_1)\cos FPG=b^2(f+f_1)/\sqrt{(b^4+a^2e^2y^2)}=(f+f_1)/\sqrt{(rr_1)}.$$
$$X=f_1\cos EPF_1+f\cos EPF=f_1(ae+x)/r_1-f(ae-x)/r.$$
$$Y=f_1\cos DPF_1+f\cos DPF=y(f_1r+fr_1)/rr_1.$$

Substituting in (1) and (2),

$$-ab^2mv^2x=rr_1(arx+a^2er-bx)f_1+rr_1(ar_1x-a^2er_1-bx)f\ \ldots\ldots\ldots\ldots\ldots\ (3).$$